QUIZ 7 SOLUTIONS: LESSONS 6-8
FEBRUARY 4, 2019

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Find the particular solution to

$$\frac{dy}{dx} = -\frac{2x^4}{y^2}; \text{ given } y = 5 \text{ when } x = 1.$$

We separate variables:

$y^2\,dy = -2x^4\,dx$

$\Rightarrow \int y^2\,dy = \int -2x^4\,dx$

$\Rightarrow \frac{1}{3}y^3 = -\frac{2}{5}x^5 + C$ or $C' = 3C$

$\Rightarrow y^3 = -\frac{6}{5}x^5 + C$

$\Rightarrow y = \sqrt[3]{-\frac{6}{5}x^5 + C}$ ← general solution

$y = 5, x = 1$

$5 = \sqrt[3]{-\frac{6}{5}(1)^5 + C}$

$5^3 = 125 = -\frac{6}{5} + C$

$125 + \frac{6}{5} = C$

$\Rightarrow C = \frac{625}{5} + \frac{6}{5} = \frac{631}{5}$

$$\boxed{y = \sqrt[3]{-\frac{6}{5}x^5 + \frac{631}{5}}}$$

2. [5 pts] Find the solution to

$$\frac{dy}{dx} = \frac{\cos(11x)}{e^{11y}}.$$

Again, we separate variables:

$e^{11y}\,dy = \cos(11x)\,dx$

$\Rightarrow \int e^{11y}\,dy = \int \cos(11x)\,dx$

Via integration by substitution,

$\frac{1}{11}e^{11y} = \frac{1}{11}\sin(11x) + C$

$e^{11y} = \sin(11x) + \underbrace{C}_{\text{or } C' = 11C}$

To undo e, we apply $\ln$ to both sides:

$\ln(e^{11y}) = \ln(\sin(11x) + C)$ ← C is inside the $\ln$

$\underbrace{\ln(e^{11y})}_{11y} = \ln(\sin(11x) + C)$

$\Rightarrow 11y = \ln(\sin(11x) + C)$

$$\Rightarrow \boxed{y = \frac{1}{11}\ln(\sin(11x) + C)}$$